Evaluating the voice capacity of 802.11 WLAN under distributed control

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***Abstract*— Though initially designed for data transport, there is increasing interest in using the 802.11 WLAN protocol for voice and other real time services. This paper presents a performance model for voice over WLAN under distributed control. The model is based on classical decoupling arguments and allows an analytical evaluation of network capacity in terms of tuneable protocol parameters. We determine parameter settings that maximize the number of simultaneous conversations and demonstrate the significant impact on capacity of the voice packet size. Models are developed for both dedicated and integrated networks using the priority features of 802.11e.**

I. Introduction

There is developing interest in using 802.11 WLANs as a support for voice communication. Demand arises in a number of specialized environments (hospitals, warehouses, factories,...) where mobility is key and infrastructure costs need to be minimized. More generally, the increasing availability of terminals with VoIP and WiFi capabilities is in itself a significant driving force for VoWLAN deployment. While experiments have shown that a limited number of conversations can be successfully sustained, there comes a point when voice and possible concurrent data load attains a saturation limit beyond which no useful voice communication is possible. The objective of the present paper is to develop an analytical model allowing us to explore this limit and to derive optimal protocol settings that maximize realized voice capacity.

Although the Point Coordination Function (PCF) was explicitly developed to provide necessary quality of service to applications like voice, the absence of widespread deployment leads us to focus our analysis on the voice capacity of the Distributed Coordination Function (DCF). We envisage the use of standard codecs (G.711 and G.729) over WLANs of nominal capacities 11 Mbps and 54 Mbps (802.11b and 802.11g, respectively). The aim is to evaluate the impact on performance of the protocol parameters governing the operation of the DCF backoff mechanism. For the integration of voice with data transport, we account for the use of differentiated DCF parameters as envisaged under the enhanced protocol, 802.11e. An additional key parameter is the voice packet size. We evaluate the capacity gains that can be realized by extending the packetization interval used in voice codecs.

Our modelling approach is to generalize the familiar decoupling arguments introduced by Bianchi [1]. The decoupling approximation greatly facilitates analysis through the assumption that the backoff status of individual stations evolves independently. The accuracy of this approximation is confirmed by our simulation results. To evaluate the performance of an integrated voice/data network, we follow the approach of Robinson and Randhawa [2] which is itself a generalization of the Bianchi analysis.

The voice capacity of 802.11 is receiving increasing attention in the literature.

Garg and Kappes [3] report on an experimental set-up where they observe the performance of an 802.11b network using a variety of codecs. With default G.711 codec and DCF parameter settings, they observe a capacity limit of 6 conversations. An approximate analytical model is proposed which notably illustrates the importance of choosing a voice packet size that is as large as possible.

Anjum et al. [4] report an experimental study of voice data integration on an 802.11b WLAN with stations transmitting at 11 Mbps. Their results show that 5 voice calls can coexist with 2 Mbps of background data; when there is no data, the voice capacity increases to 8 conversations.

Elaoud and Agrawal [5] also perform an experimental evaluation of VoWLAN. They consider both packet loss and packet delay performance metrics. Loss constraints alone would allow 10 conversations on the 802.11b network. Voice capacity decreases to 6 or 4 conversations when taking account of delay constraints appropriate for LAN or WAN communications, respectively. The paper also includes an approximate analytical evaluation.

Hole and Tobagi [6] propose a simple analytical model for the voice capacity of an 802.11b WLAN using various codecs and packet sizes. The analysis assumes no collisions and thus leads to an upper bound on capacity. They also use simulations to show the effect of delay constraints on capacity.

A recent paper by Clifford et al. [7] evaluates voice capacity using an alternative adaptation of the Bianchi decoupling approach to the one proposed here. They point to possible capacity gains to be derived by giving higher priority to the access point.

Finally, in addition to the decoupling analysis of 802.11e in [2], it is worth mentioning two evaluations of this protocol by means of simulation [8] and [9].

The following sections introduce the developed analytical models and present numerical results derived from their application to particular WLAN configurations. In Section III we first derive the fixed-point equations defining the performance of a dedicated voice network. Solution of these equations allows an evaluation of packet delays and consequently a determination of voice capacity. The model is generalized in

Section IV to evaluate the performance of an integrated voice data 802.11e WLAN. In addition to voice packet delay, the analysis here provides an estimation of data throughput under the assumption that a certain number of stations always have packets to send. Section V presents a number of numerical results that highlight the impact on performance of various parameter choices. We are notably able to propose optimal settings that maximize voice capacity under dedicated and integrated network scenarios. Conclusions on optimal parameter choices and perspectives for further developments of network and model are presented in final Section VI.

II. Protocols and standards

A. IEEE 802.11 protocols

We evaluate a WLAN voice service assuming medium access is controlled using the DCF. DCF is a CSMA/CA (carrier sense multiple access - collision avoidance) protocol fully defined in [10]. We do not consider the additional RTS/CTS virtual carrier sense mechanism as this is known to be inefficient for small packets like those generated by voice codecs. The alternative PCF was specified with services like voice in mind but has so far not been widely implemented.

The main tuneable DCF parameters are the minimum contention window $CW_{\min}$ and the maximum number m of backoff stages for which this window doubles. The contention window increases after successive collisions to a maximum $CW_{\max} = 2^m CW_{\min}$. Default values are $CW_{\min} = 32$ and $m = 5$ but these can be changed, notably to provide service differentiation as recommended in the IEEE 802.11e specification [11].

The 802.11e standard also allows different inter-frame spaces $DIFS$ for different classes of service. The space is then known as $AIFS$, for arbitration inter-frame space. A station with a smaller value of $AIFS$ has first try in accessing the medium when its backoff counter authorizes an attempt.

To simplify the analysis, we assume all users experience the same radio conditions and therefore have the same transmission rate R (e.g., $R = 11$ Mbps). This assumption is not crucial and can easily be removed.

Finally, system performance strongly depends on the durations of successful transmissions T and collisions T_{col}. For example in the case of 802.11b we have:

$$T = DIFS + t^{pr} + \frac{\sigma}{R} + SIFS + t^{pr} + \frac{ack}{R},$$

$$T_{col} = DIFS + t^{pr} + \frac{\sigma}{R}.$$

The inter-frame spaces $DIFS$ and $SIFS$, and t^{pr}, the time to transmit the PLCP preamble and physical header, do not depend on the transmission rate. Default values are $DIFS = 50\mu$s, $SIFS = 10\mu$s, $t^{pr} = 192\mu$s for $R = 1$Mbps and $t^{pr} = 96\mu$s for other transmission rates. The duration of an empty slot, $SLOT$ is 20μs. The ack packet length is 14 bytes. The packet length σ includes the MAC, IP and transport headers. Assuming voice is transmitted using RTP/UDP, this overhead amounts to 74 bytes.

B. Voice coding

The performance of voice calls depends strongly on the packet size and arrival frequency, as determined by the codec used for voice packetization. For our numerical results we consider two of the most commonly used codecs, G.711 and G.729. The G711 codec packetizes 10 ms of audio into an 80 byte payload (64 Kbps). The G729 codec compresses 10 ms of audio into a 10 byte payload (8 Kbps). It is however possible to create transport packets that contain more than one audio packet, thus increasing the payload and decreasing overhead. For example, the G.711 codec may be used to transmit packets of 160 byte payload every 20ms. Increasing the payload may have undesirable effects such as increased delay and jitter. The impact of such factors is examined in Section V.

III. Voice traffic only

We first consider a WLAN cell supporting VoIP traffic only. The WLAN has a fixed number x of remote stations, each maintaining a full-duplex voice call going through an access point (AP). We aim to evaluate the maximum value of x compatible with performance requirements for voice calls. This is the voice capacity of the WLAN and is determined essentially by delays suffered by the incoming packets emitted by the AP.

The analytical model is based on the decoupling assumption introduced by Bianchi [1]. Remote stations and the AP are assumed to attain a stationary regime where their attempts to access the channel can be considered to be mutually independent. The evolution of the backoff process for each station can then be modelled as a Markov chain yielding the probability that the station attempts to use the channel in any given slot. This probability is expressed in terms of the probability a collision occurs on an access attempt. Since the collision probability can also be written in terms of the attempt probability, a set of equations is derived that can be solved by fixed-point iteration.

It can be shown that this approach is asymptotically correct as the number of stations increases [12]. It has also been validated as an approximation by comparison with simulations, even when the number of stations is small [1]. The reader is referred to [13] for a detailed study of the existence and uniqueness of the fixed point.

We must adapt the usual analysis to account for the fact that stations do not always have packets to send. We assume each remote station emits one packet every D seconds while the access point emits packets at rate x/D.

To distinguish AP and remote voice stations, we label variables with an index $i \in \{a, v\}$. Let λ_i denote the stationary probability the station is active (i.e., has a packet to transmit), p_i the probability it then attempts to use the channel in an arbitrary slot, d_i the packet transmission time, i.e., the time from when it first appears at the head of the station buffer to the moment it is successfully transmitted, and c_i the probability the station finds the channel busy when it makes an attempt.

Note that p_i is determined from the same Markov chain defined in the analysis of a saturated station [1]. The probability that the station transmits in any slot is $\lambda_i p_i$. The quantities p_i, λ_i, d_i, c_i for $i = a$ and v thus satisfy the following equations:

$$c_v = 1 - (1-\lambda_a p_a)(1-\lambda_v p_v)^{x-1}, \quad (1)$$

$$c_a = 1 - (1-\lambda_v p_v)^{x}, \quad (2)$$

$$p_i = F(c_i), \quad i = v, a, \quad (3)$$

$$\lambda_v = \mathrm{E}[d_v]/D, \quad (4)$$

$$\lambda_a = x\mathrm{E}[d_a]/D, \quad (5)$$

where the function F is given by [1]:

$$F(c) = \frac{2(1-2c)}{(CW_{\min}+1)(1-2c) + CW_{\min} c(1-(2c)^m)}.$$

The values λ_i can be interpreted as the load of the transmit queue at a station of type i.

Packet transmission times can be expressed as:

$$d_i = \sum_{k=1}^{K_i} \sum_{n=1}^{U^k} S_i^{k,n} + (K_i - 1)T_{col} + T,$$

where K_i is the number of channel attempts for the given packet, U^k is a random variable uniformly distributed on $[0, 2^{(k-1)\wedge m} CW_{\min} - 1]$ and $S_i^{k,n}$ is the duration of slot n as seen by the given type i station during its backoff phase k.

The probability a packet takes k attempts is $Pr[K_i = k] = (1-c_i)c_i^{k-1}$ and we deduce:

$$\begin{aligned} \mathrm{E}[d_i] &= \mathrm{E}_i[S] \frac{CW_{\min}}{2} \left[\frac{1-(2c_i)^m}{1-2c_i} + \frac{(2c_i)^m}{1-c_i} \right] \\ &\quad + \frac{c_i}{1-c_i} T_{col} + T, \end{aligned}$$

with the mean slot durations $\mathrm{E}_i[S]$ as follows:

$$\mathrm{E}_v[S] = (1-c_v)SLOT + \mathrm{P}_v(x-1)T + (c_v - \mathrm{P}_v(x-1))T_{col} \,,$$

$$\mathrm{E}_a[S] = (1-c_a)SLOT + \mathrm{P}_a(x)T + (c_a - \mathrm{P}_a(x))T_{col} \,,$$

where $\mathrm{P}_v(x-1)$ is the probability of a successful transmission by one of the other $x-1$ remote stations or the access point, as seen by the given remote voice station:

$$\mathrm{P}_v(x) = x\lambda_v p_v (1-\lambda_v p_v)^{x-1}(1-p) + (1-\lambda_v p_v)^x p,$$

and $\mathrm{P}_a(x)$ is the probability of a successful transmission by one of the x remote stations, as seen by the access point:

$$\mathrm{P}_a(x) = x\lambda_v p_v (1-\lambda_v p_v)^{x-1}.$$

Assuming a voice buffer of one packet, delay in the remote stations is $d_v \wedge D$ (generally, we find $d_v << D$). To evaluate delay in the access point we model the joint arrival process from the x conversations as a Poisson process. This is a simple conservative approximation that appears satisfactory for present purposes. The expected packet delay, $\mathrm{E}[\delta_a]$, is then:

$$\mathrm{E}[\delta_a] = \mathrm{E}[d_a] + \frac{x\mathrm{E}[d_a^2]}{2(1-\lambda_a)D},$$

where the expression for $\mathrm{E}[d_a^2]$ is given in the appendix.

The voice capacity is strictly determined by a remote percentile of the delay distribution (e.g., probability of packet delay exceeding 100 ms should be less than 10^{-3}). In fact, it turns out that delays are typically very small until the number of stations attains a value where the AP is completely saturated. The above formulas are used in Section V to evaluate the performance of typical configurations.

IV. Differentiated services

We now extend our model to analyse the performance of an 802.11e WLAN offering differentiated services. To the WLAN of Section III we add a fixed number y of saturated remote data stations. We could also consider data transmission from the AP although a slight change in the model would be required to account for the additional data traffic and the change in AP packet transmission time. Differentiation is thus between stations and not within stations.

Classes of traffic are differentiated by varying $CW_{\min}$, m, and $AIFS$. Delay-sensitive voice traffic can be given priority by assigning these parameters smaller values than those for data traffic. Let $CW_{\min}^j$, m_j, and $AIFS_j$ be the parameters for traffic class j, with $j = 1$ for voice and $j = 2$ for data. Following IEEE 802.11e recommendations [11], we set $AIFS_2 = AIFS_1 + l \times SLOT$ for some integer l. Following [2] and [13], we analyse this system by considering two contention zones: zone A where only stations carrying class 1 can attempt to use the channel, i.e., the $(l-1)SLOT$s following an idle $AIFS_1$, and zone B where stations carrying any class may transmit, i.e., the remaining $SLOT$s.

In addition to the notation of Section III, let p_d denote the probability a data station attempts to use the channel at the start of a zone-B slot and let c_d be the probability that station then sees a busy channel leading to a collision. Writing $q_A = (1-\lambda_a p_a)(1-\lambda_v p_v)^x$ and $q_B = (1-p_d)^y q_A$, the stationary probabilities of being in zone A or B are, respectively:

$$\pi_A = \frac{1 + q_A + \cdots + q_A^{l-1}}{1 + q_A + \cdots + q_A^{l-1} + q_A^l/(1-q_B)},$$

$$\pi_B = \frac{q_A^l/(1-q_B)}{1 + q_A + \cdots + q_A^{l-1} + q_A^l/(1-q_B)}.$$

The respective probabilities of seeing a busy channel are:

$$\begin{aligned} c_v &= \pi_A \left(1 - (1-\lambda_a p_a)(1-\lambda_v p_v)^{x-1}\right) \\ &\quad + \pi_B \left(1 - (1-\lambda_a p_a)(1-\lambda_v p_v)^{x-1}(1-p_d)^y\right) \end{aligned} \quad (6)$$

$$\begin{aligned} c_a &= \pi_A \left(1 - (1-\lambda_v p_v)^x\right) \\ &\quad + \pi_B \left(1 - (1-\lambda_v p_v)^x (1-p_d)^y\right) \end{aligned} \quad (7)$$

$$c_d = (1 - (1-\lambda_a p_a)(1-\lambda_v p_v)^x (1-\lambda_d p_d)^{y-1}). \quad (8)$$

The channel attempt probabilities are now:

$$p_d = F_2(c_d), \quad (9)$$

$$p_i = F_1(c_i), \quad i = v, a, \quad (10)$$

with

$$F_j(c) = \frac{2(1-2c)}{(CW_{\min}^j + 1)(1-2c) + CW_{\min}^j c(1-(2c)^{m_j})}.$$

Relations (4), (5), (6-10) form a system of equations that can be solved by fixed-point iteration. To calculate voice and data performance given the respective attempt and collision probabilities is then straightforward though the formulas are quite long. These quantities depend on mean slot durations as seen by each station type, as shown in the appendix.

The AP load is $x\mathrm{E}[d_a]/D$. When this is less than 1 we estimate average packet delay using the M/G/1 model as in Section III.

The mean throughput of data traffic is given by:

$$\gamma_d = \frac{\sigma_d \pi_B y p_d (1-p_d)^{y-1}(1-\lambda_a p_a)(1-\lambda_v p_v)^x}{\pi_A \mathrm{E}[S_A] + \pi_B \mathrm{E}[S_B]},$$

where σ_d is the data packet length, $E[S_A]$ and $E[S_B]$ are the mean durations of slots in zone A and B, respectively. The appendix gives their expressions together with other formulas used to derive the numerical results presented in the next section.

V. Results

We first consider voice capacity in a WLAN cell with only voice stations and then investigate the impact of data traffic.

A. Dedicated voice network

Figure 1 compares analysis and simulation through the expected AP packet delay in a WLAN where all users transmit at 11 Mbps and use the G.711 codec. We use default DCF values $CW_{\min} = 32$ and $m = 5$. The results are typical of those derived for other configurations and confirm the accuracy of the analytical model. Note that voice capacity is here determined by the AP load limit: delays are typically very small until the number of conversations is such that the AP is saturated.

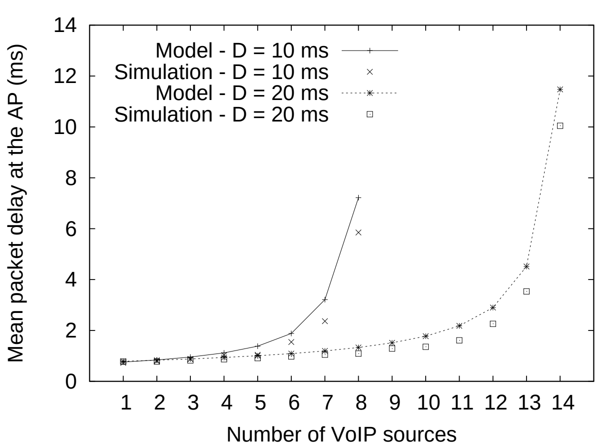

Fig. 1. Mean packet delay, DCF default parameters.

Voice capacity is rather small for the standard G.711 inter-packet time of 10 ms. This is because of the excessive overhead constituted by packet headers and the incompressible protocol timings $DIFS$, $SIFS$ and t^{pr}. Reducing the coding rate (i.e., using G.729) and increasing the channel capacity (i.e., using the 802.11g rate of 54 Mbps) has only a slight impact on capacity. The voice capacity attained in these configurations is given in Table I.

A more significant gain in capacity is obtained by increasing the packet size. By doubling or tripling the packetization delay, the payload is increased in the same proportions leading to a much lower relative overhead. The voice capacity of an 11 Mbps WLAN is almost doubled when D is increased to 20 ms (see Fig. 1). Further results are shown in Table I. Clearly, this gain comes at the cost of increased delay and jitter. However, as shown in Fig. 1, queueing delay does not increase unduly and the impact of the increased packetization delay D is slight.

TABLE I

Maximum number of simultaneous voice calls

11Mbps

G711 D(ms)	$(CW_{\min}, m)$ (32,5)	(8,0)
10	8	10
20	14	18
30	19	23

G729 D(ms)	$(CW_{\min}, m)$ (32,5)	(8,0)
10	9	12
20	17	23
30	26	35

54Mbps

G711 D(ms)	$(CW_{\min}, m)$ (32,5)	(8,0)
10	10	14
20	19	27
30	28	39

G729 D(ms)	$(CW_{\min}, m)$ (32,5)	(8,0)
10	10	14
20	20	29
30	30	43

The results in Table I also highlight the significant impact of the settings of $CW_{\min}$ and m. Figure 2 shows the gain in capacity as first $CW_{\min}$ is reduced from 32 to 8 and then m is reduced from 5 to 0. The shorter backoff intervals have the effect of reducing the amount of time wasted by stations that wait too long before attempting to access the channel. In our experiments, the values $CW_{\min} = 8$ and $m = 0$ appear to be optimal over the range of tested WLAN configurations. To reduce $CW_{\min}$ further tends to degrade performance due to increased collisions.

B. Performance of voice data integration

Figure 3 shows the voice capacity with the integration of data services, assuming default 802.11b parameters of $CW_{\min}$, m, and $AIFS$. The data sources are assumed to generate packets of size 1000 bytes. The addition of data stations significantly reduces voice capacity: adding a single data station reduces the voice capacity by more than a third. Note that even though the load at the AP still limits the voice capacity, the mean packet delay at the AP can be quite significant at this limit, reaching over 50ms in some cases. In the case of voice data integration, considering the AP packet delay as a performance metric may slightly reduce the voice capacity.

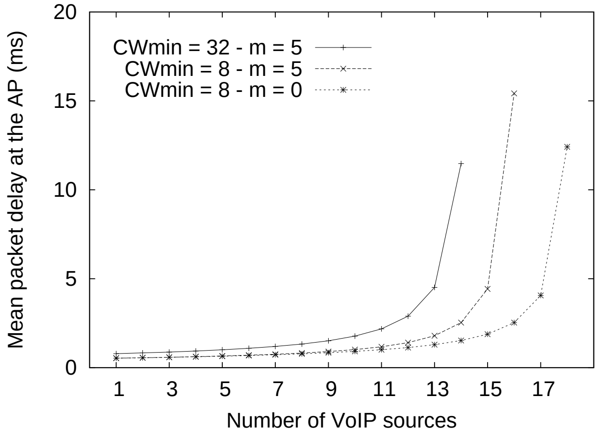

Fig. 2. Impact of $CW_{\min}$, m (G.711, D=20ms).

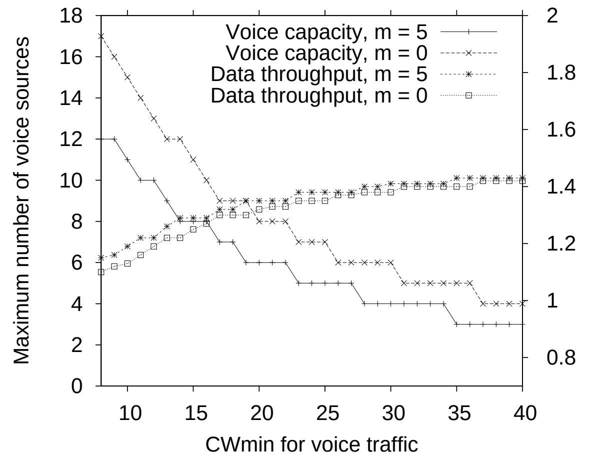

Fig. 4. Impact of $CW^1_{\min}$, m_1, default parameters for data.

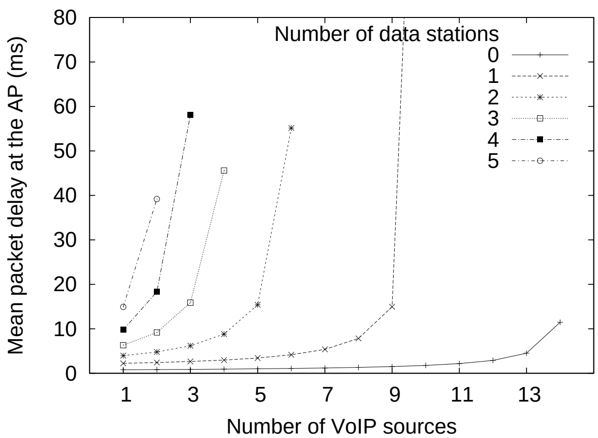

Fig. 3. Integration of voice and data.

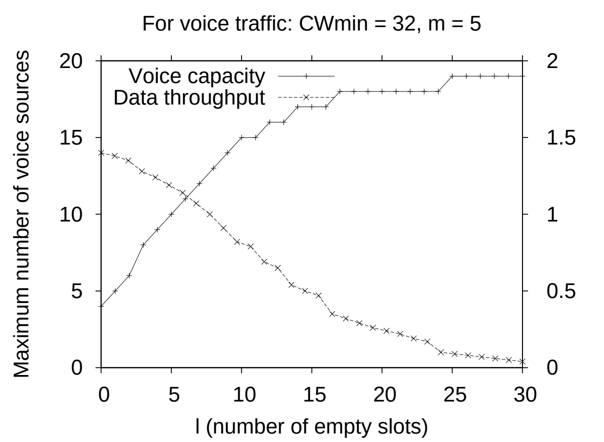

Fig. 5. Impact of $AIFS$, $AIFS_2 = AIFS_1 + l \times SLOT$.

In Figure 4 we fix the number of data stations to 3 and study the effectiveness of varying $CW_{\min}$ and m for voice stations as recommended in 802.11e to improve voice capacity. Indeed, decreasing $CW^1_{\min}$ to the value of 8, triples the voice capacity, while reducing the data throughput by only 20%. Setting m_1 to 0 further increases the voice capacity with negligible reduction in data throughput.

In Figures 5 and 6 we investigate the impact of varying $AIFS$. We observe that even higher gains in voice capacity may be achieved by varying this parameter rather than $CW^1_{\min}$ and m_1. However these gains come at the expense of significant reduction in data throughput, to the extent that no resources are available for data traffic when the difference between $AIFS_1$ and $AIFS_2$ approaches $CW^1_{\min} \times SLOT$.

VI. Conclusion

The analytical model developed in Section III confirms the disappointingly low voice capacity of the IEEE 802.11 DCF WLAN under default parameter settings, known from experiments and simulations. It is possible, however, to considerably augment capacity by more appropriate choices. Packet size has a significant impact and can advantageously be increased by doubling or tripling the codec packetization delay. Voice capacity is further increased by reducing the initial backoff window $CW_{\min}$ from 32 slots to 8 and inhibiting window expansion ($m = 0$).

The same parameter settings for voice stations appear close to optimal in the case of an integrated voice data network when data stations use default values. The presence of stations emitting data packets significantly reduces voice capacity, even when voice traffic has priority. Voice performance can be further protected by the choice of $AIFS$ parameters. However the difference in $AIFS$ between traffic classes must be kept small, i.e. between 1 and 5 slots, in order to prevent starvation of the lower priority traffic.

This work may be extended to include ad hoc configurations and more realistic data traffic models (TCP traffic, downlink data, etc.). The model may then be used to develop new protocol features and traffic engineering mechanisms such as admission control.

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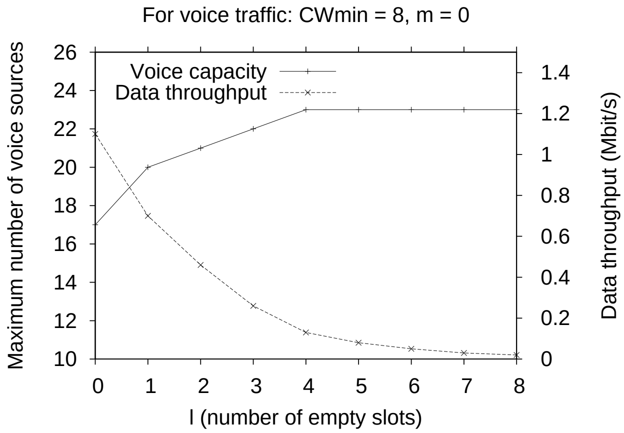

Fig. 6. Impact of $AIFS$, $AIFS_2 = AIFS_1 + l \times SLOT$.

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APPENDIX

The second moment of the packet transmission delay at the AP, used in Section III is as follows:

$$\begin{aligned}
\mathrm{E}[d_a^2] &= \mathrm{E}_a^2[S]CW_{\min}^2(1-c_a)\left[\frac{2(1-c_a^m)}{9(1-c_a)} - \frac{1-(2c_a)^m}{1-2c_a}\right. \\
&+ \frac{10(1-(4c_a)^m)}{9(1-4c_a)} + \frac{c_a^m(52^{2m}-182^{m-1}+4)}{18(1-c_a)} \\
&+ \left.(\frac{2^{2m}}{3} - 2^{m-1})\frac{c_a^m}{(1-c_a)^2} + \frac{2^{2m-3}c_a^{m+1}}{(1-c_a)^3}\right] \\
&+ T_{col}^2\frac{c_a(1+c_a)}{(1-c_a)^2} + T^2 + \mathrm{E}_a[S]CW_{\min}T_{col}\cdot \\
&\left[\frac{3c_a-7c_a^2+4c_a^3}{(1-c_a)^2(1-2c_a)^2}\right. \\
&+ \left.\frac{(2c_a)^{m+1}c_a(3-m)+(2c_a)^m(c_a(3m-4)-m)}{2(1-c_a)^2(1-2c_a)^2}\right] \\
&+ \mathrm{E}_a[S]CW_{\min}T\frac{1-c_a(1+(2c_a)^{m-1})}{(1-2c_a)(1-c_a)} \\
&+ 2T_{col}T\frac{c_a}{1-c_a}.
\end{aligned}$$

We now give the modified mean slot durations as seen by the stations (used in Section IV) as a function of T_j^i and $T_{j,col}^i$ given by:

$$T_j^i = AIFS_j + t^{pr} + \frac{\sigma_i}{R} + SIFS + t^{pr} + \frac{ack}{R},$$
$$T_{j,col}^i = AIFS_j + t^{pr} + \frac{\sigma_i}{R}.$$

$$\begin{aligned}
\mathrm{E}_v[S] &= (1-c_v)SLOT \\
&+ \pi_B\mathrm{P}_d(y)(1-\lambda_a p_a)(1-\lambda_v p_v)^{x-1}T_1^d \\
&+ (\pi_A\mathrm{P}_v(x-1)+\pi_B(1-p_d)^y\mathrm{P}_v(x-1))T_1^v \\
&+ \alpha T_{1,col}^d \\
&+ \big(1-(1-\lambda_a p_a)(1-\lambda_v p_v)^{x-1}-\mathrm{P}_v(x-1)\big)\cdot \\
&\quad (\pi_A+\pi_B(1-p_d)^y)T_{1,col}^v\,,
\end{aligned}$$

$$\begin{aligned}
\mathrm{E}_a[S] &= (1-c_a)SLOT\pi_B\mathrm{P}_d(y)(1-\lambda_v p_v)^x T_1^d \\
&+ (\pi_A\mathrm{P}_v(x)+\pi_B(1-p_d)^y\mathrm{P}_v(x))T_1^v \\
&+ \beta T_{1,col}^d \\
&+ \big(1-(1-\lambda_v p_v)^{x-1}-\mathrm{P}_v(x)\big)\cdot \\
&\quad (\pi_A+\pi_B(1-p_d)^y)T_{1,col}^v\,,
\end{aligned}$$

where α and $beta$ are such that the probabilities of all events (i.e. $SLOT$, $T_{j,col}^i$, etc.) sum to 1, and

$$\mathrm{P}_d(y) = yp_d(1-p_d)^{y-1}.$$

The mean slot duration in zones A and B are as follows:

$$\begin{aligned}
E[S_A] &= SLOT\,[1-(1-\lambda_a p_a)(1-\lambda_v p_v)^x] \\
&+ T_1\,[\lambda_a p_a(1-\lambda_v p_v)^x \\
&+ x\lambda_v p_v(1-\lambda_a p_a)(1-\lambda_v p_v)^{x-1}] \\
&+ T_{1,col}\,[\lambda_a p_a(1-(1-\lambda_v p_v)^x) \\
&+ x\lambda_v p_v(1-(1-\lambda_a p_a)(1-\lambda_v p_v)^{x-1})]
\end{aligned}$$

$$\begin{aligned}
E[S_B] &= SLOT\,[1-(1-\lambda_a p_a)(1-\lambda_v p_v)^x(1-p_d)^y] \\
&+ T_1(1-p_d)^y\,[\lambda_a p_a(1-\lambda_v p_v)^x \\
&+ x\lambda_v p_v(1-\lambda_a p_a)(1-\lambda_v p_v)^{x-1}] \\
&+ T_2 yp_d(1-\lambda_a p_a)(1-\lambda_v p_v)^x(1-p_d)^{y-1} \\
&+ T_{1,col}(1-p_d)^y\,[\lambda_a p_a(1-(1-\lambda_v p_v)^x) \\
&+ x\lambda_v p_v(1-(1-\lambda_a p_a)(1-\lambda_v p_v)^{x-1})] \\
&+ T_{2,col}\times\zeta,
\end{aligned}$$

where ζ is such that the probabilities of all events sum to 1.