AUCTION-BASED BANDWIDTH ALLOCATION A CROSS-ENTROPY APPROACH

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EXTENDED ABSTRACT

Bandwidth allocation in a market-based context can be performed with the aim of maximizing one among (or a combination of) different objective functions, e.g. the efficient use of network resources (use as much bandwidth as possible, maximizing the total revenues) or the customer satisfaction (admit as much users as possible).

According to the network portion spanned by the bandwidth request, this objective may be searched at a global level (so that there is unique intra-domain objective function), or at a local level (where each operator optimizes the objective function within its domain, so that there are different inter-domain objective functions).

The Generalized Vickrey algorithm (GVA) has been proposed as a mean to implement a bandwidth market, through a demand-based dynamic pricing scheme [1]. However, the optimization of any objective function, be it on a global or on a local scale, represents a combinatorial maximization problem, raising scalability concerns as the number of users grows and therefore endangering the viability of such an allocation scheme.

For this purpose we consider a simultaneous one round (multi unit) Vickrey auction where the i-th user submits a connection request characterized by the 4-tuple $(n_{i1}, n_{i2}, w_i, b_i)$, i.e.:

The IDs of the origin an destination nodes, supposed to be associated by a single path

The bandwidth requirement w_i (in the simplest case all users require the same bandwidth, i.e. $w_i = 1$

The bid b_i, which is supposed to be drawn from a probability distribution (e.g. the uniform or the exponential), reflecting the dispersion of the subjective values assigned by the single users to the bandwidth under assignment

In this context user is a generic term to describe any bandwidth request. We are specifically dealing with interconnection of Autonomous Systems (AS), assuming that any Autonomous System has a number of interconnection points with its neighboroughing ASs, so that each AS represents generally a very large number of bandwidth requests, i.e. of bids, and therefore a number of users (each bid referring to a specific destination). The network hosting the bandwidth requests is considered to be resource-limited.

Two different scenarios can be considered for carrying out the bids: a) a local one where each AS just dialogues with its neighboroughing ASs, which, if accepting the bid for destination outside their domain, assume the responsibility to carry the traffic up to the final destination bidding in turn to the next AS on the path; b) a global one where each AS dialogues with all the ASs along the path to destination bidding to each of them for their part. In the first phase we think of focussing on the local approach only.

Under this formulation any solution to the bandwidth allocation problem can be expressed by a binary N-vector $\underline{v} = [v_1, v_2, \ldots, v_N]$, N being the number of users. If the corresponding objective function is $T(\underline{v})$ the allocation goal is to look for the vector $\underline{v}^* = \max_{\underline{v}} T(\underline{v})$ under the constraint that the capacity of each link is not exceeded, i.e.

$$\sum_{(n_{i1}, n_{i2}) \in l} w_i < C_l \quad \forall l$$

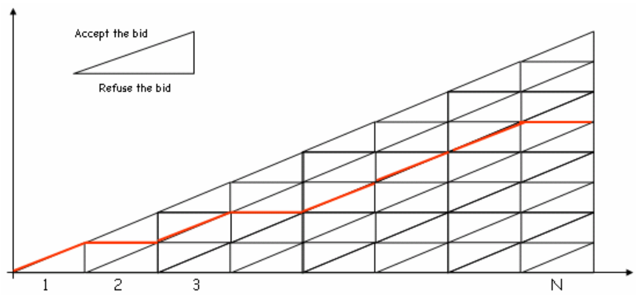

Figure 1 – Search for optimal solution as path in the bid domain

The allocation solution can be graphically represented as a path moving sequentially through the bids by either a flat (bid refused) or ascending (bid accepted) trait, as in Figure 1, where the optimal path is marked in red. Here the number of candidate solutions is 2^N, i.e. grows exponentially with the number of bids, so that the exhaustive search may be impractical

We can resort to an adaptive combinatorial optimization technique such the cross-entropy method, which has been already successfully employed in a number of rare event estimation and combinatorial optimization problems.

The starting point of the analysis is the intuition that, in a revenue maximization problem, the optimal bid allocation strategy $\underline{v}^*$ lies in the vicinity a bid-per-unitary-bandwidth strategy v_s, up to network resources saturation. The optimal path is therefore tackled in an adaptive fashion through a suitable convergence criterion.

The use of the cross-entropy method allows to transform the combinatorial optimization problem in an estimation one through the use of an indicator function. The solution of the associated estimation problem provides us with a peaky probability density function which actually highlights, as the most probable path indicated by the pdf, the path (i.e. the bandwidth allocation) maximizing the objective function associated with the optimization problem. The estimation problem is of easier solution than the original combinatorial optimization one since the results of Importance Sampling theory are used in conjunction with the use of the Kullback-Leibler (cross-entropy) distance.

In order to define a mathematical framework for the problem formulation, let's consider the space X of possible solutions represented by the set of all possible bid combinations. For a set of N bids each element of this space is a binary n-uple $\underline{x} = (x_1, x_2, \ldots, x_N)$ where $x_i = 1$ if the i-th bid is accepted and 0 otherwise. The general maximization problem aims at identifying the vector $\underline{x}^* = \arg\max_{x \in X} S(x)$ maximizing our objective function $S(x)$.

The resulting combinatorial optimization problem can be associated with an estimation one through the use of an indicator function $I(x,\gamma)$ detecting the crossing of a threshold γ by our objective function:

$$I(x,\gamma) = \begin{cases} 0 & S(x) < \gamma \\ 1 & S(x) \geq \gamma \end{cases}$$

If the objective function had the maximum value γ^*, the indicator function $I(x,\gamma^*)$ would return that maximum location. If we now consider a family $f(x,p)$ of discrete probability function on X parameterized by a real valued parameter (vector) $\underline{p}$. the stochastic problem associated to the optimization one is

$$l_p(\gamma) = P[S(x) > \gamma]$$

which can be solved by simulation. Since typically $l_p(\gamma) = 1/|X|$, $|X|$ being the cardinality of X, which can be very high, the combinatorial optimization problem turns to a rare event estimation problem. The long simulation time due to the event rarity can be slashed by resorting to the Importance Sampling approach, where the initial uniform distribution is replaced by a biased distribution that enhances the frequency of important events (Importance Sampling), and compensates the bias through the multiplication by the likelihood ratio. The critical point, i.e. the choice of the biased density function can in turn be solved by the cross-entropy approach introduced by Rubinstein [2] [3], which replaces the a priori choice of the biased density by an adaptive approach where there is a continuous cycle of small simulation runs and re-tuning of the biased density.

REFERENCES

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